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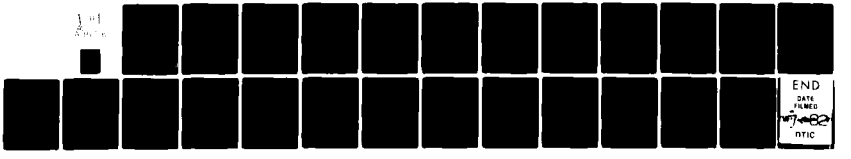

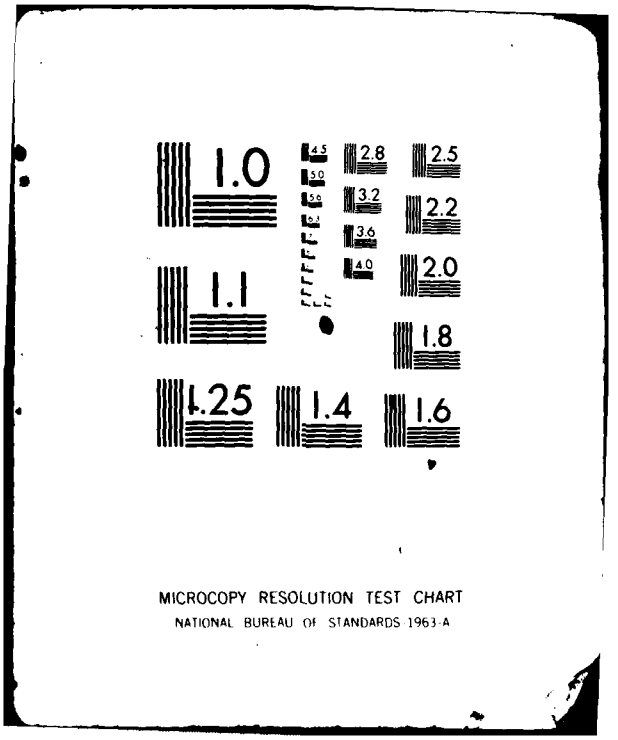
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AN EXAMPLE OF HOMOGENIZATION IN LINEAR THERMOELASTICITY

BY

Gilles A. Francfort

SUDAM Report No. 82-1

May 1982

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OF
APPLIED
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ENGINEERING

STANFORD
UNIVERSITY
STANFORD,
CALIFORNIA
94305

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ASYMPTOTIC ANALYSIS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This study is a sequel of Francfort [1], where homogenization of linear dynamic thermoelasticity with rapidly varying coefficients had been performed. We demonstrate how consideration of a simple one-dimensional example leads to numerical results that exhibit all the features that were theoretically predicted and in particular the presence of fast oscillations in the temperature field.

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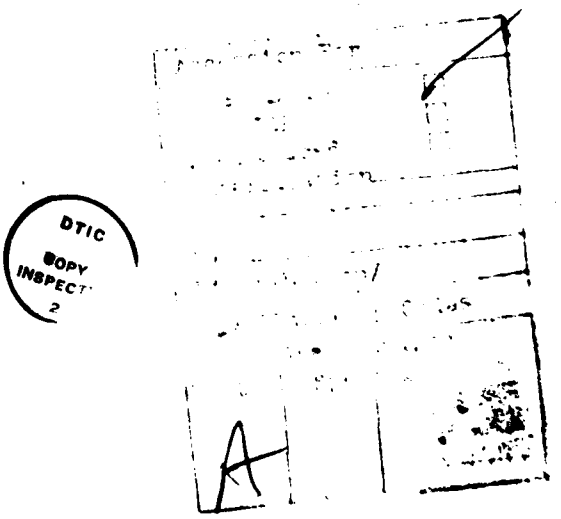
ABSTRACT

This study is a sequel of Francfort [1], where homogenization of linear dynamic thermoelasticity with rapidly varying coefficients had been performed. We demonstrate how consideration of a simple one-dimensional example leads to numerical results that exhibit all the features that were theoretically predicted and in particular the presence of fast oscillations in the temperature field.



INTRODUCTION

In a previous report (see Francfort [1]) we studied the homogenization of an idealized, thermoelastic composite material. The idealization resided in the assumed periodicity of all relevant mechanical and thermal coefficients. We established the existence of an equivalent homogeneous thermoelastic material. The solution of any initial-boundary value problem for a body made of this composite material was found to converge to the solution of a related initial-boundary value problem for a body made of the equivalent material as the size of the period of the coefficients approaches zero. In the latter problem, the boundary conditions, the initial displacement and velocity fields remain unchanged, but a change in the initial temperature field takes place. Such a change is highly unusual in this type of problems. We then proceeded to analize the phenomena that accompany such a change. Using asymptotic expansions, we disclosed the existence of fast oscillations in the temperature field. These oscillations are generated by the periodic structure of the material, although they need not be periodic. The temperature field of the equivalent homogeneous material represents a time average of these fast oscillations.

In this report we illustrate these phenomena with a one-dimensional example. The presence of fast oscillations in the temperature field, as well as the necessity of the shift in initial temperature are clearly evidenced.

1. Description of the Problem

We consider a bar of length L made of a periodic assembly of two different materials. This bar is to be studied in tension-compression. It is built-in and both ends are at ambient temperature.

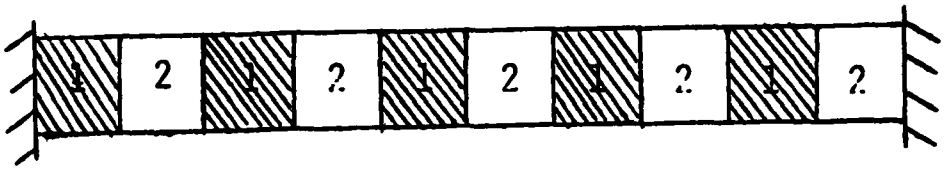


Figure 1

The two thermoelastic media that enter the problem are denoted by 1 and 2, respectively. Note that this bar can be thought of as an infinite, layered strip provided both ends are built-in and are maintained at constant temperature and provided the displacements are taken to be normal to the layering.

Our reference cell Y is taken to be as in Fig. 2.

and its length is taken to be unity.

We define all relevant coefficients on Y by:

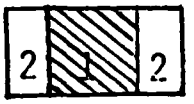


Figure 2

$$\left.\begin{aligned} E(y) &= \begin{cases} E_1 & \text{for material 1} \\ E_2 & \text{for material 2} \end{cases}, \text{ Young's modulus,} \\ &\alpha(y) \quad \text{, coefficient of thermal expansion} \\ &\rho(y) \quad \text{, density} \\ &T_0\lambda(y) \quad \text{, coefficient of thermal conductivity} \\ &\frac{T_0\beta(y)}{\rho(y)} \quad \text{, specific heat,} \end{aligned}\right\} \tag{1-1}$$

T_0 being the reference temperature.

Then if n is the number of cells that compose the bar, each of these cells is the ε-scaled version of Y, where

$$\varepsilon = \frac{L}{n} \tag{1-2}$$

Our set of equations can be written as

$$\begin{aligned} \rho(x/\varepsilon)\ \ddot{u}^{\varepsilon} &= \frac{d}{dx}\left(E(x/\varepsilon)\left(\frac{du^{\varepsilon}}{dx} - \alpha(x/\varepsilon)\tau^{\varepsilon}\right)\right) \\ \beta(x/\varepsilon)\ \dot{\tau}^{\varepsilon} &= \frac{d}{dx}\left(\lambda(x/\varepsilon)\ \frac{d\tau^{\varepsilon}}{dx}\right) - E(x/\varepsilon)\ \alpha(x/\varepsilon)\ \frac{d^2u^{\varepsilon}}{dtdx} \quad , \end{aligned} \tag{1-3}$$

where $u^{\varepsilon}_{(x,t)}$ is the displacement field, $\tau^{\varepsilon}_{(x,t)}$ is the increment of temperature field, and a dot denotes time differentiation. The boundary conditions associated with (1.3) are

$$u^{\varepsilon}(0,t) = u^{\varepsilon}(L,t) = 0 \ ; \ \tau^{\varepsilon}(0,t) = \tau^{\varepsilon}(L,t) = 0 \tag{1-4}$$

We then impose on the bar a set of initial conditions:

$$u^{\varepsilon}(x,0) = f(x) \ ; \ \dot{u}^{\varepsilon}(x,0) = g(x) \ ; \ \tau^{\varepsilon}(x,0) = k(x) \tag{1-5}$$

We propose to compare the solution of (1-3)-(1-5) to the solution of the related initial-boundary value problem for the equivalent homogeneous material. In our case the homogenized coefficients can be explicitly computed with the help of Eqns (1-27) of Francfort [1].

We obtain

$$E = 2\frac{E_1E_2}{E_1 + E_2} \quad , \quad \lambda = \frac{2\lambda_1\lambda_2}{\lambda_1 + \lambda_2} \, , \, \alpha = \frac{\alpha_1 + \alpha_2}{2} \, , \, \bar{\rho} = \frac{\rho_1 + \rho_2}{2} \, ,$$

$$\bar{\beta} = \frac{\beta_1 + \beta_2}{2} \, , \, \gamma = \frac{(E_1\alpha_1 - E_2\alpha_2)(E_1 - E_2)}{2(E_1 + E_2)} \, , \, \sigma = \frac{(E_1\alpha_1 - E_2\alpha_2)^2}{2(E_1 + E_2)} \, ,$$

$$\beta = \bar{\beta} + \sigma \tag{1-6}$$

Then the equations for the equivalent material are given by:

$$\bar{\rho}\ddot{u} = E\frac{d^2u}{dx^2} - E\alpha\frac{d\tau}{dx} \, , \, \beta\dot{\tau} = \lambda\frac{d^2\tau}{dx^2} - E\alpha\frac{d^2u}{dtdx} \tag{1-7}$$

where u and τ are, respectively, the displacement and the increment of temperature fields. The boundary conditions remain unchanged, i.e.:

$$u(0,t) = u(L,t) = 0 \; ; \; \tau(0,t) = \tau(L,t) = 0 \tag{1-8}$$

The initial conditions are:

$$u(x,0) = f(x) \, , \, \dot{u}(x,0) = g(x) \, , \, \tau(x,0) = \frac{\bar{\beta}k(x) + \gamma\frac{df}{dx}(x)}{\beta} \tag{1-9}$$

Since we are in no position, even with very simple initial conditions, to solve either problem exactly, we have to resort to a numerical procedure which is briefly described in the next section.

2. Discretization of the Problem

We first perform a space discretization of the equations (1-3) and (1-7). Our discretization consists of taking piecewise linear shape functions. Having performed the standard steps of a finite element analysis, i.e. Galerkin and matrix formulations, we obtain the following system of ordinary differential equations:

$$M\ddot{d} = -Kd - H\dot{\tau} \quad , \quad \tilde{M}\dot{\tau} = -\tilde{K}\tau - \tilde{H}\dot{d} \tag{2-1}$$

where M, K, H, $\tilde{M}$, $\tilde{K}$, $\tilde{H}$ are suitable consistent mass, stiffness and damping matrices and where the vectors d, τ are, respectively, the displacement and the temperature vectors. We do not specify which system equations (2-1) are referring to since we discretize (1-3) and (1-7) in a similar manner. Of course the values of all the relevant matrices will be different in each case.

To solve (2-1) we rewrite it as a second-order system in d and define η as

$$\eta = \int_0^t \tau \, dt \tag{2-2}$$

We obtain

$$\begin{bmatrix} M & 0 \\ 0 & \tilde{M} \end{bmatrix} \frac{d^2}{dt^2} \begin{vmatrix} d \\ \eta \end{vmatrix} + \begin{bmatrix} 0 & H \\ \tilde{H} & \tilde{K} \end{bmatrix} \frac{d}{dt} \begin{vmatrix} d \\ \eta \end{vmatrix} + \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix} \begin{vmatrix} d \\ \eta \end{vmatrix} = 0 \tag{2-3}$$

and we use on (2-3) a fully implicit trapezoidal Newmark algorithm (see Zienkiewicz [6]). Such an algorithm insures unconditional stability and second order accuracy.

The actual finite element program used is Taylor's FEAP (finite element analysis program; refer to Zienkiewicz [6]),modified and improved by J. Wingett [5].

3. Specific Problem Treated and Numerical Results

Caution should be exercised in a transient analysis when realistic data are considered, since the relative magnitudes of the coefficients have several consequences:

i) since the damping factor in the equations is rather small, attenuation will take place on a very long time scale; a short time scale is, however, necessary to accurately reproduce the displacement field,

ii) the magnitude of the coefficients strains the numerical procedure because it overextends the precision capabilities of the computer.

This is why, as a first step, we consider unrealistic coefficients that will not generate such complications but that will clearly illustrate the diverse phenomena occurring in the problem. We suppose that all relevant coefficients are equal to 1. for the first material, and to 2. for the second. The initial conditions are of two different types: initial temperature with no initial displacements or velocities (case (a)) or initial displacements with no initial temperature or velocities (case (b)). In both cases, we take these initial conditions to be of the form

$$1 - \cos \frac{2\pi x}{L}$$

Our choice of coefficients implies a reduction of the initial temperature by a factor of 2 in case (a) when passing from a heterogenous to a homogenous material. The computations should show unequivocally that this shift is necessary, because, in view of the linearity of the solution with respect to the initial conditions, a temperature twice as large initially will produce a homogenized temperature and displacement field twice as large for all subsequent

times. Such a solution should then totally disagree with the solution for the heterogenous medium. This is confirmed by Figures 3 to 6 which give the displacement and temperature field of a point located at one-third of the bar as a function of time:

- Figures 3 and 4 correspond to 7.5 cells
- Figures 5 and 6 correspond to 40.5 cells

The fast temperature oscillation as foreseen in Francfort [1] is clearly present. One should observe that the homogeneous temperature averages the fast oscillations.

Figures 7 to 10 illustrate the behavior of the fields at the same point for case (b). The fast oscillations are less obvious for 7.5 cells, but appear clearly when 40.5 cells are considered.

Figures 11 and 12 are a space representation of the displacement and temperature fields at $t = 1s$, with 40.5 cells present. In the case of the temperature field, one should not expect a close agreement between both curves: the fast oscillations in time make the heterogenous solution oscillate about the homogeneous one; thus at a given time, we merely obtain a "stroboscopic" picture of this oscillation.

We now turn our attention to a more realistic composite, made of epoxy and graphite. The relevant data are taken from Miner-Sandstone [4] for graphite, and C. Lynch [3] for epoxy. For such values of the coefficients, σ is negligible but $\gamma/\bar{\beta}$ is -1.81. The need for a non-zero initial temperature for the homogeneous material due to a non-zero initial displacement should be clearly felt. We consider the initial data to be a initial displacement of the form

$$u(0) = 10^{-3}\left(1 - \cos\frac{2\pi x}{L}\right) \quad \text{(ft.)} \tag{3.3}$$

Coefficients	Epoxy	Graphite
Density ρ in lb/ft^3 (kg/m^3)	72(1153.)	120 (1922.)
Young's modulus E in lb/ft^2 (N/m^2)	7(335.3) $10^7(10^7)$	32 (1532.7) $10^7(10^7)$
Thermal exp. coef. α in $°F^{-1}$ ($°C^{-1}$)	3.3 (5.9) $10^{-5}(10^{-5})$	.21 (.38) $10^{-5}(10^{-5})$
Specific heat C_e in ft/°F (m/°C)	350 (191.1)	128.1 (70.3)
Thermal conduct. λ in lb/s °F (W/m°C)	.0434 (.355)	.6482 (5.310)
Reference temper. T_0 in °F (°C)	70 (21)	70 (21)

Figures 13 and 14 do show the need for a non-zero initial temperature in the case of a non-zero initial displacement. The presence of fast oscillations which average in time to the homogenized temperature field is unquestionable.

Note that the number of cells considered in this last problem is relatively high (70.5 cells). Note also that no damping has occurred on the scale of our computations. This confirms our preliminary remarks.

It is appropriate at this point to stress that the solution of a thermo-elastic problem differs radically from the solution of an elastic, or heat conduction problem. The bumps that can be observed in Fig. 13 could never appear in a purely elastic displacement field, nor could the sharp angles of Fig. 14 in a pure heat conduction problem.

All the computations of Section 3 were performed on an IBM 3033 with an ORVYL compiler. The average CPU time needed for 160 space points and 500

iterations is 90s. Consideration of the number of operations involved in the different steps of a two-dimensional problem shows that CPU times of the order of four hours would be necessary to perform a similar computation. The associated cost would be clearly prohibitive under most circumstances.

CONCLUSION

In this study we have demonstrated the reality of the initial temperature shift and its link to the existence of fast oscillations in temperature. As predicted in Francfort [1], the shift is necessary as soon as the initial "phase" of the oscillation is different from zero, which is the case in the examples treated.

The frequencies of the fast oscillations are not the natural vibration frequencies of a cell. They are given through (2.20) of Francfort [1] and, as remarked there, the oscillations need not be periodic, even when only a finite number of frequencies of the cell are excited (see Francfort [2], Part I, Ch. 3, Section 4 for an example.)

An experimental confirmation of the theoretical and numerical results obtained here would be a fascinating and worthwhile task.

ACKNOWLEDGMENT

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REFERENCES

1. G. Francfort, "Homogenization and Linear Thermoelasticity", Stanford University Division of Applied Mechanics Report No. 81-4, August 1981.

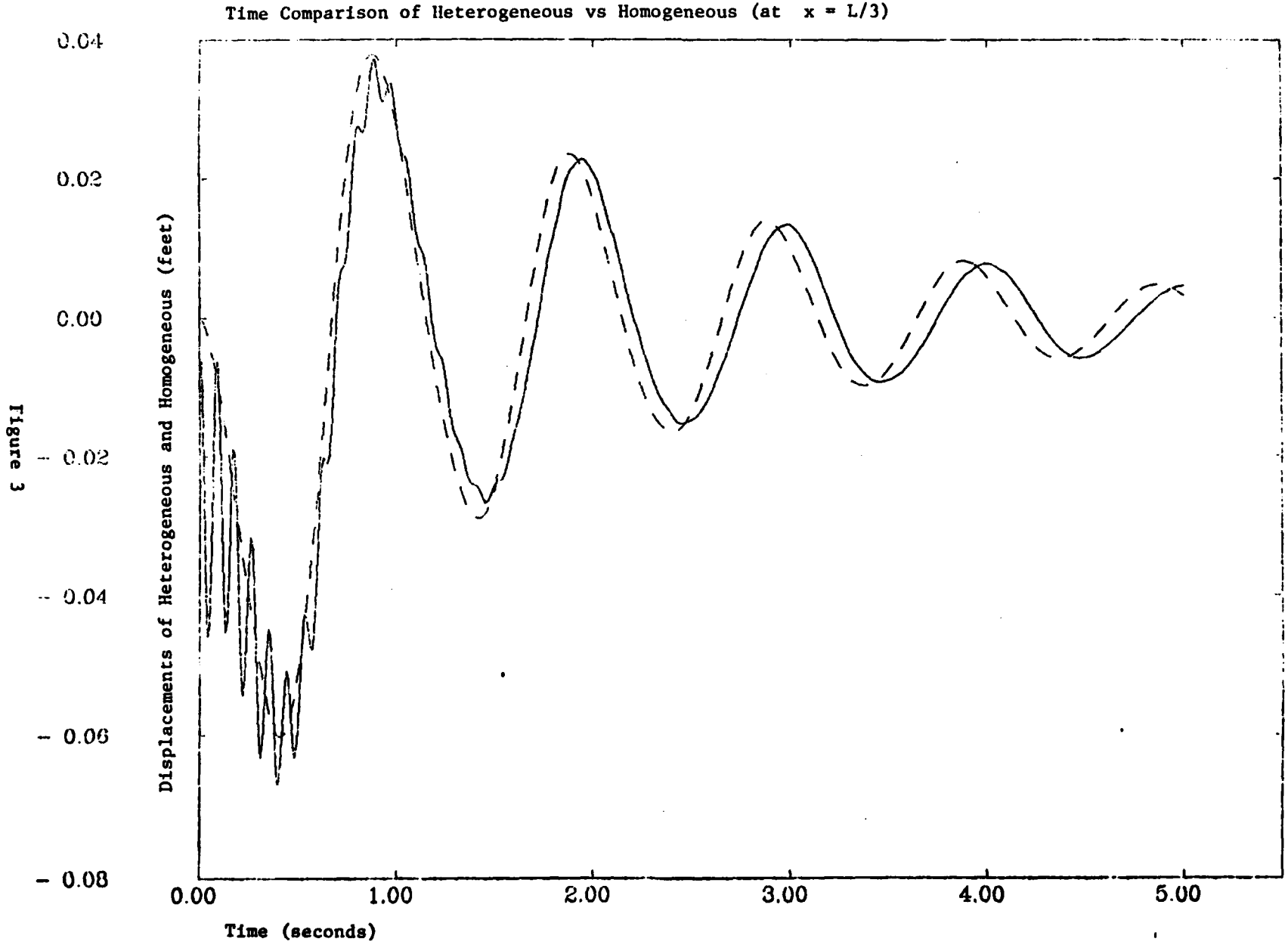
2. G. Francfort, Ph.D. Dissertation, Stanford University, April 1982.

3. C. Lynch, C.R.C.,Handbook of Material Science, Vol. III, C.R.C. Press 1975.

4. Miner and Sandstone, Handbook of Materials, J. Wiley, 1955.

5. J. Wingett, private communication.

6. O.C. Zienkiewicz, The Finite Element Method, 3rd Edition, McGraw Hill, 1977.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0.04
0.02
0.00
- 0.02
- 0.04
- 0.06
- 0.08
Displacements of Heterogeneous and Homogeneous (feet)
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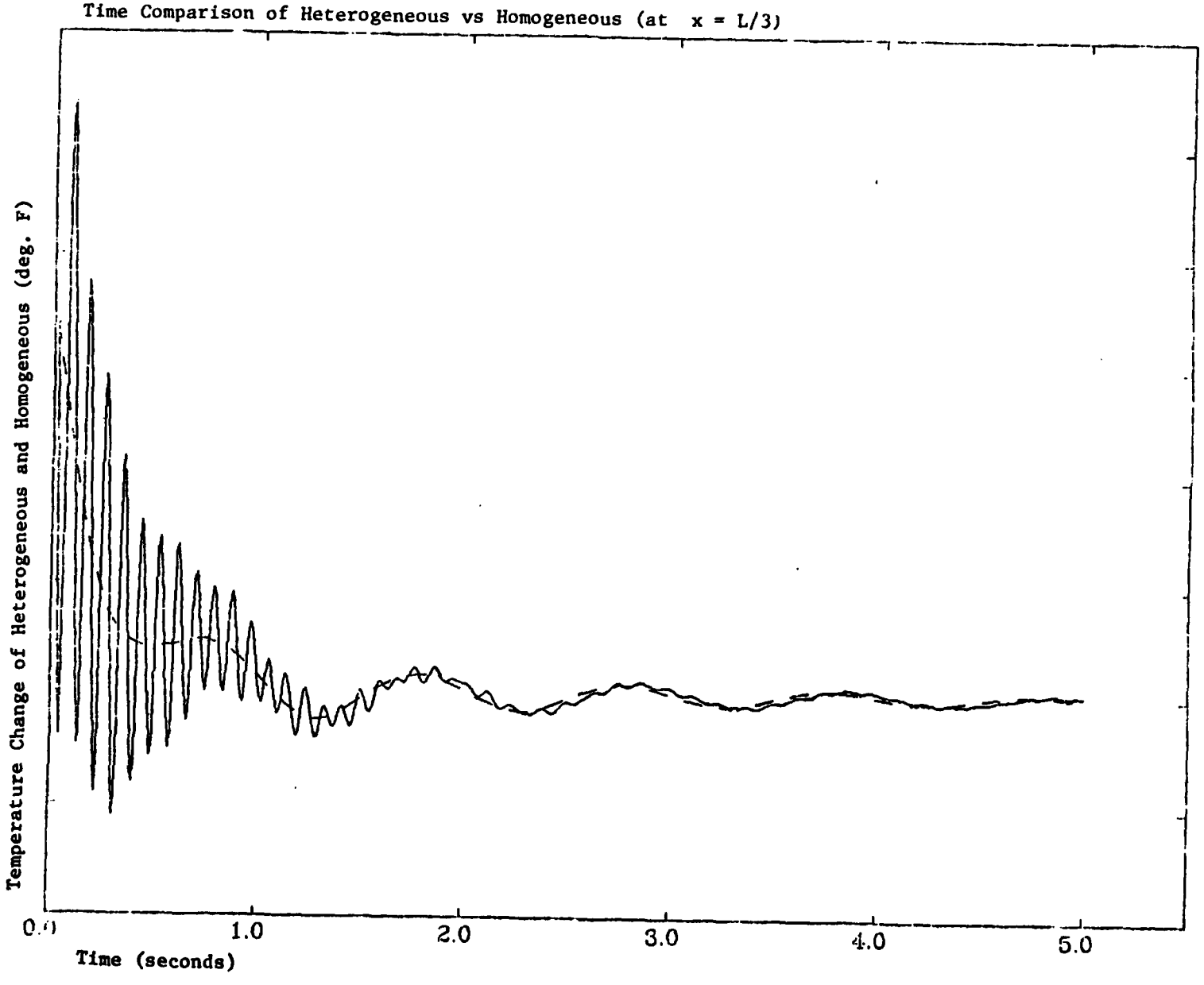
1.00
2.00
3.00
4.00
5.00
Time (seconds)

Figure 3


Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
Temperature Change of Heterogeneous and Homogeneous (deg. F)
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0.8
0.6
0.4
0.2
0.0
- 0.2
- 0.4
0.0
1.0
2.0
3.0
4.0
5.0
Time (seconds)

Figure 4

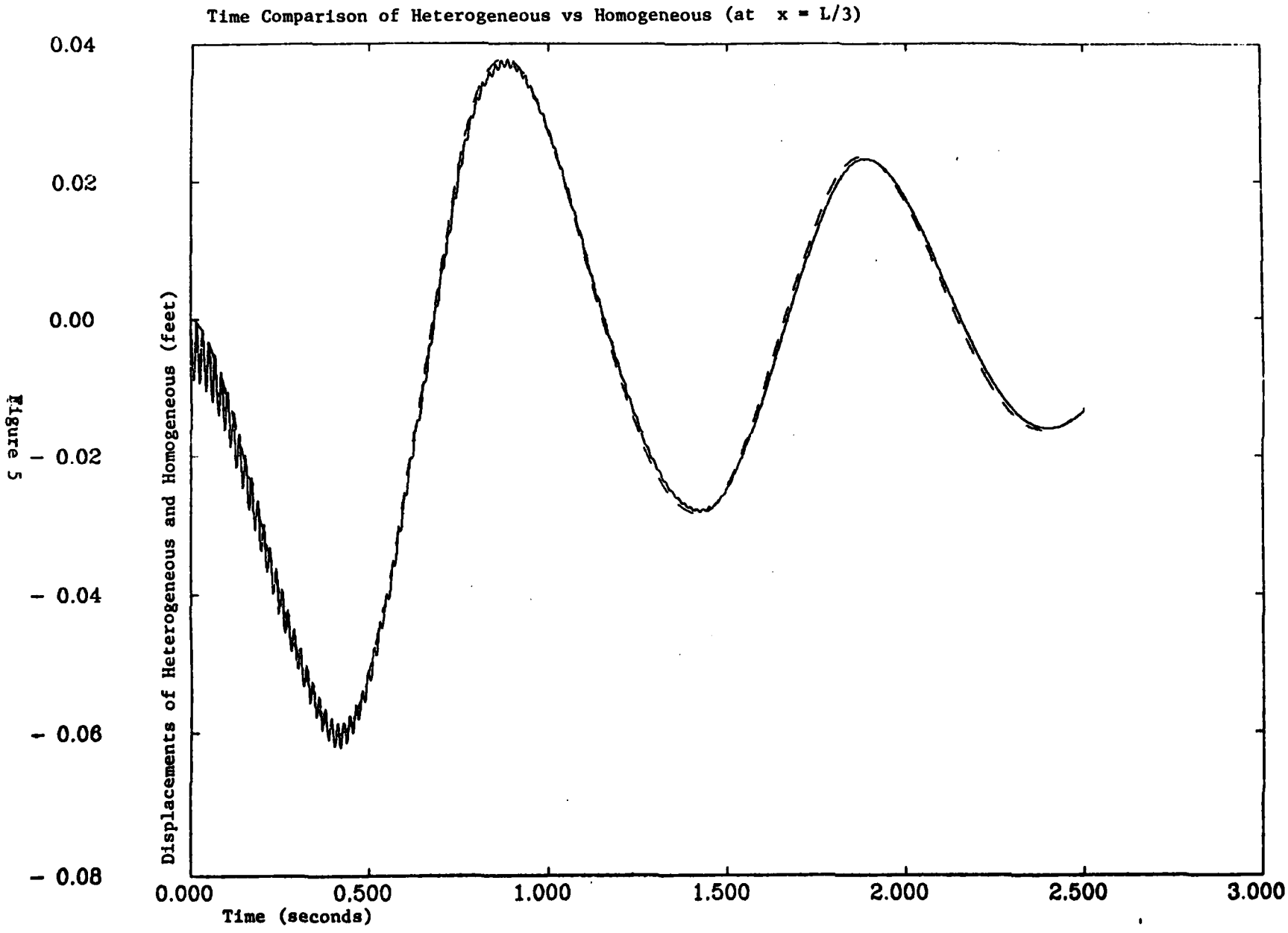
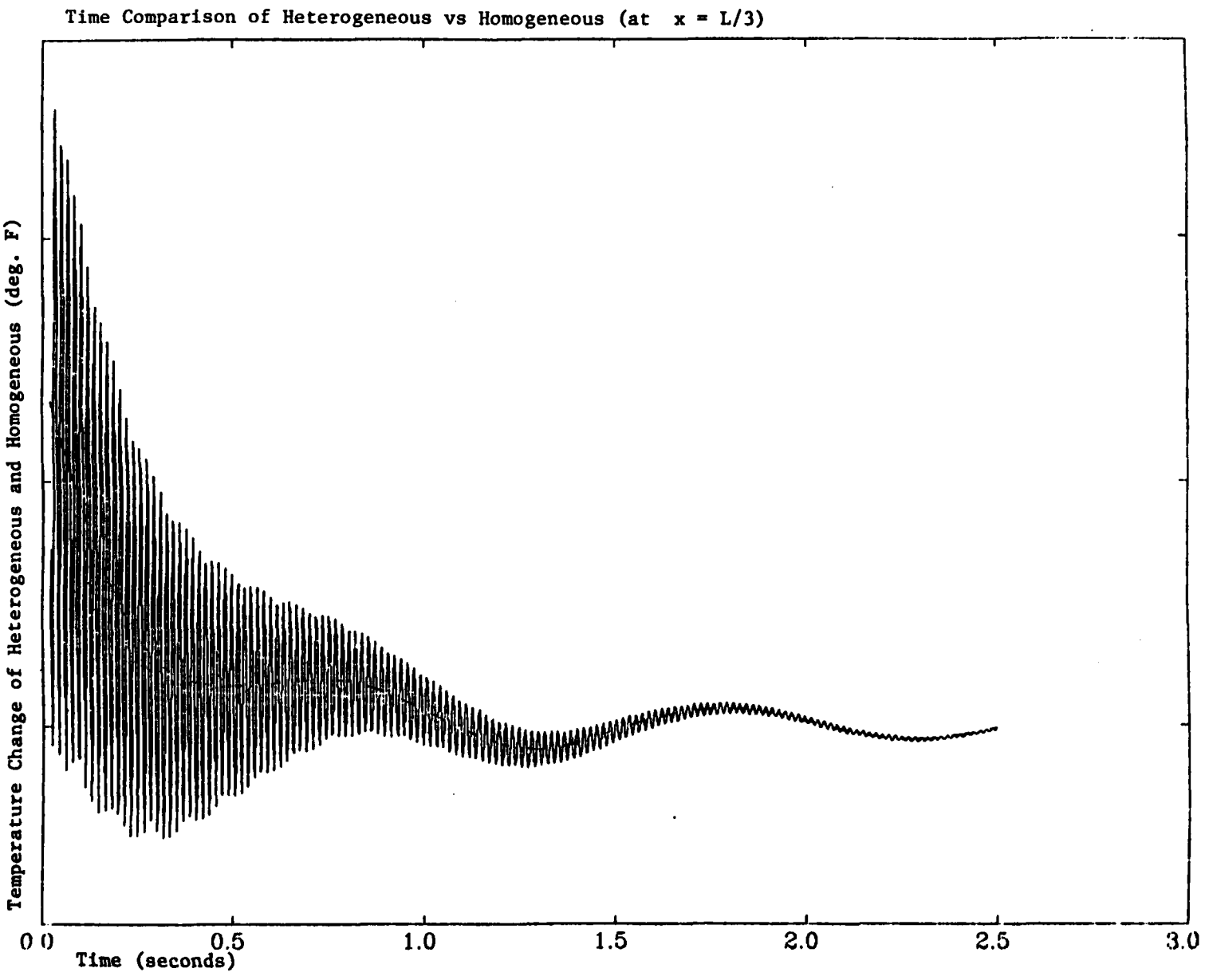

Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
0.04
0.02
0.00
- 0.02
- 0.04
- 0.06
- 0.08
Displacements of Heterogeneous and Homogeneous (feet)
0.000
0.500
1.000
1.500
2.000
2.500
3.000
Time (seconds)

Figure 5


Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
Temperature Change of Heterogeneous and Homogeneous (deg. F)
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0.0
0 0
0.5
1.0
1.5
2.0
2.5
3.0
Time (seconds)

Figure 6

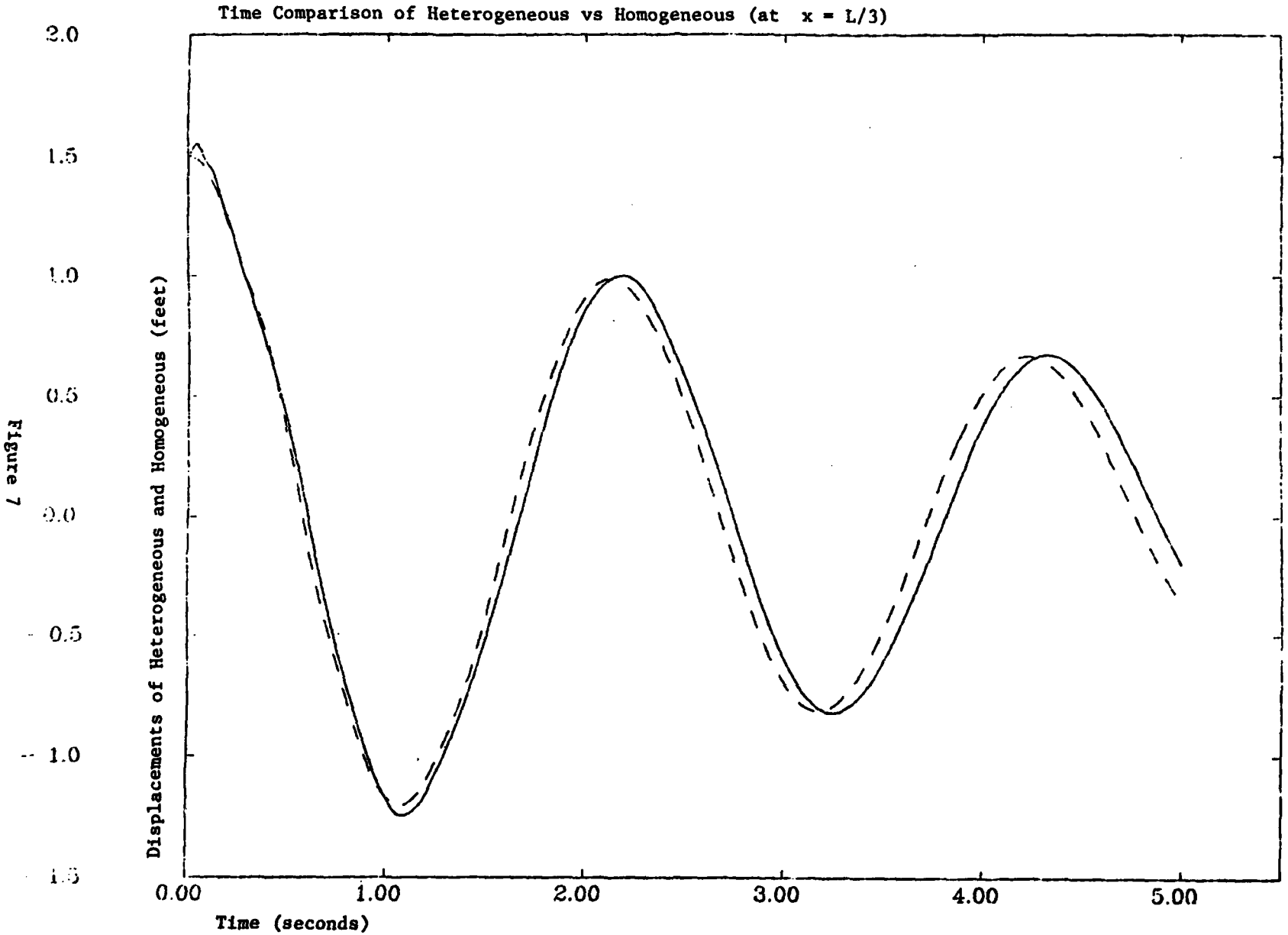
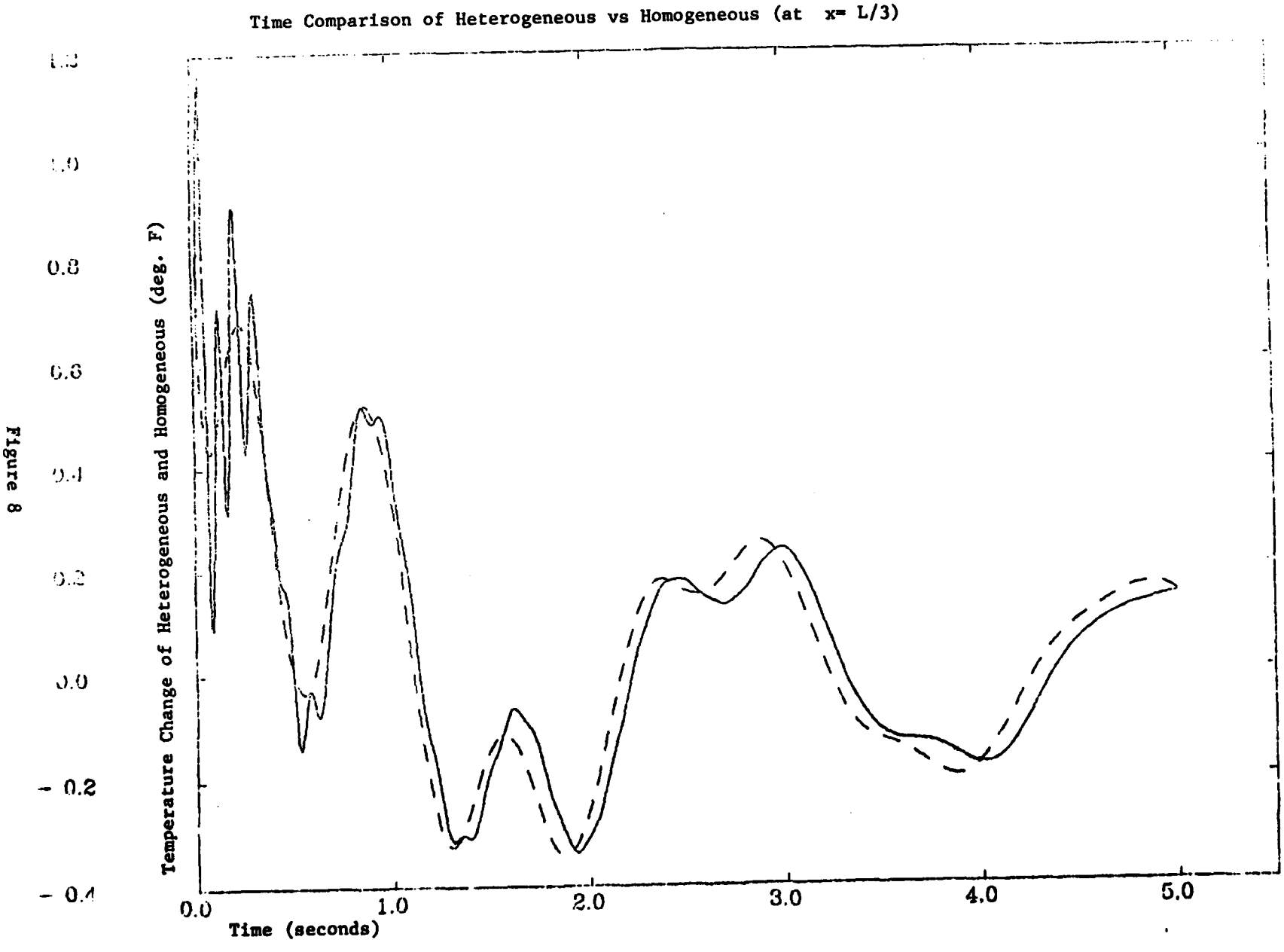

Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
2.0
1.5
1.0
0.5
0.0
- 0.5
- 1.0
- 1.5
Displacements of Heterogeneous and Homogeneous (feet)
0.00
1.00
2.00
3.00
4.00
5.00
Time (seconds)

Figure 7


Time Comparison of Heterogeneous vs Homogeneous (at x= L/3)
Temperature Change of Heterogeneous and Homogeneous (deg. F)
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0.8
0.6
0.4
0.2
0.0
- 0.2
- 0.4
0.0
1.0
2.0
3.0
4.0
5.0
Time (seconds)

Figure 8

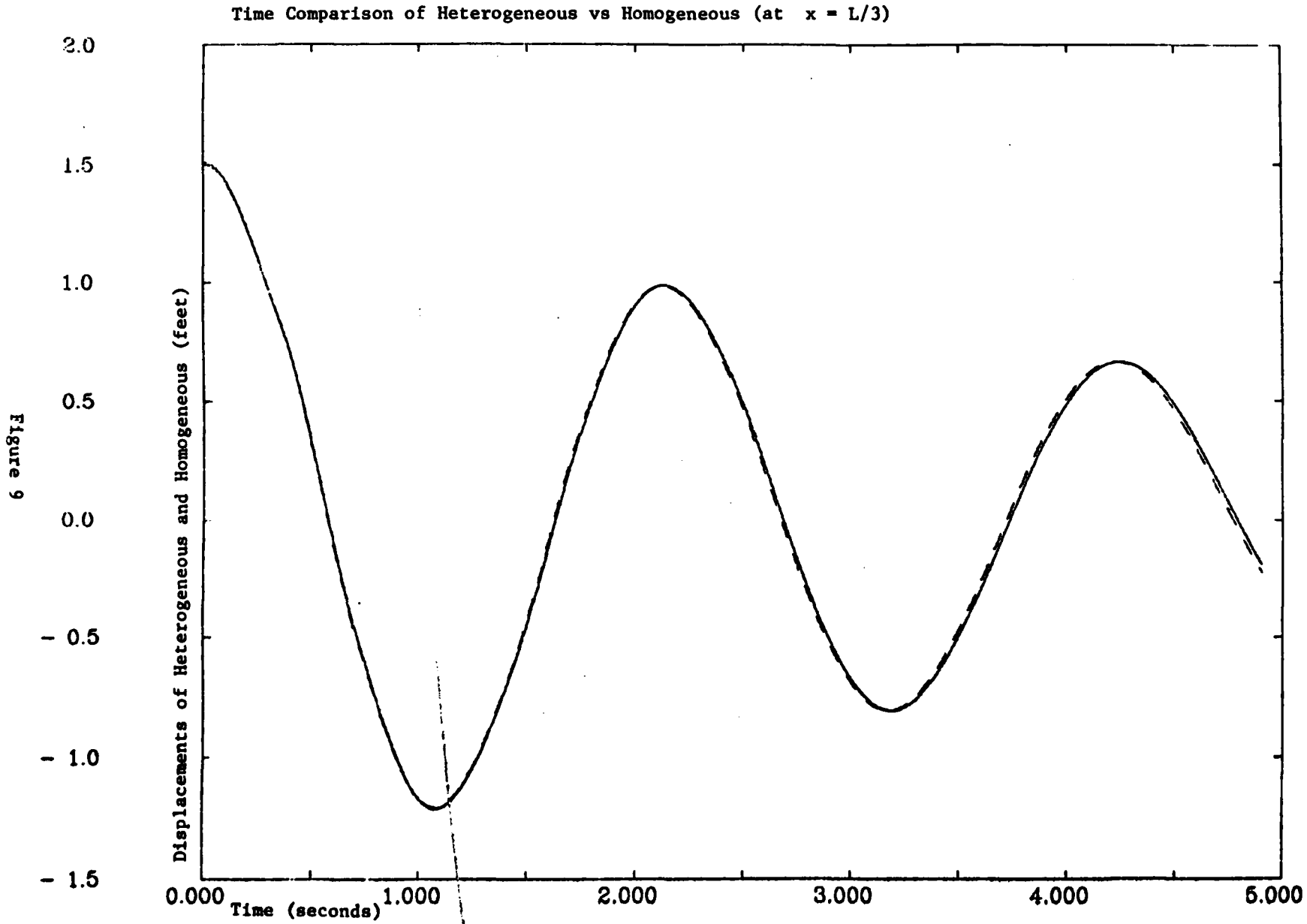
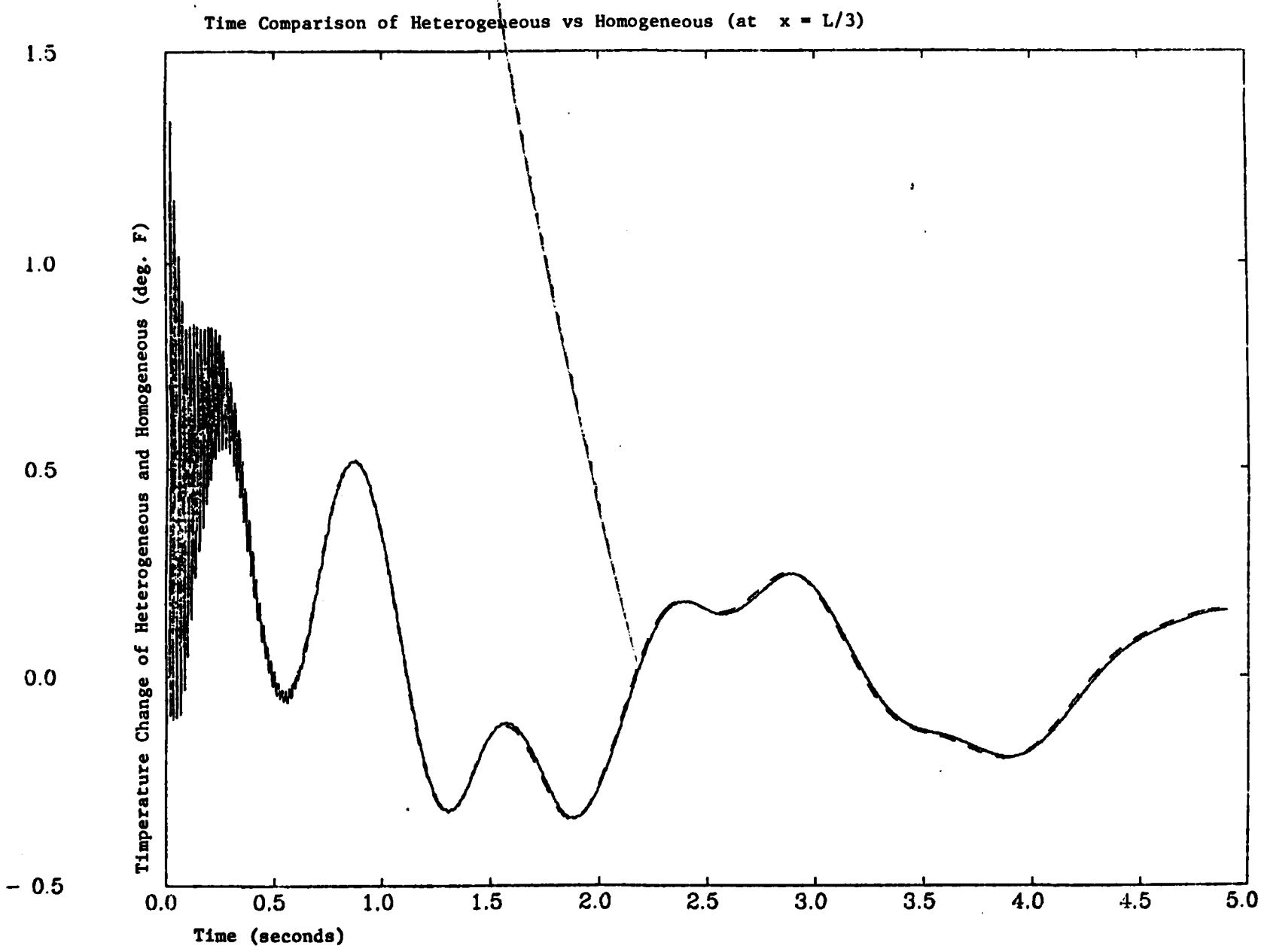

Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
2.0
1.5
1.0
0.5
0.0
- 0.5
- 1.0
- 1.5
Displacements of Heterogeneous and Homogeneous (feet)
0.000
1.000
2.000
3.000
4.000
5.000
Time (seconds)

Figure 9


Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
Timperature Change of Heterogeneous and Homogeneous (deg. F)
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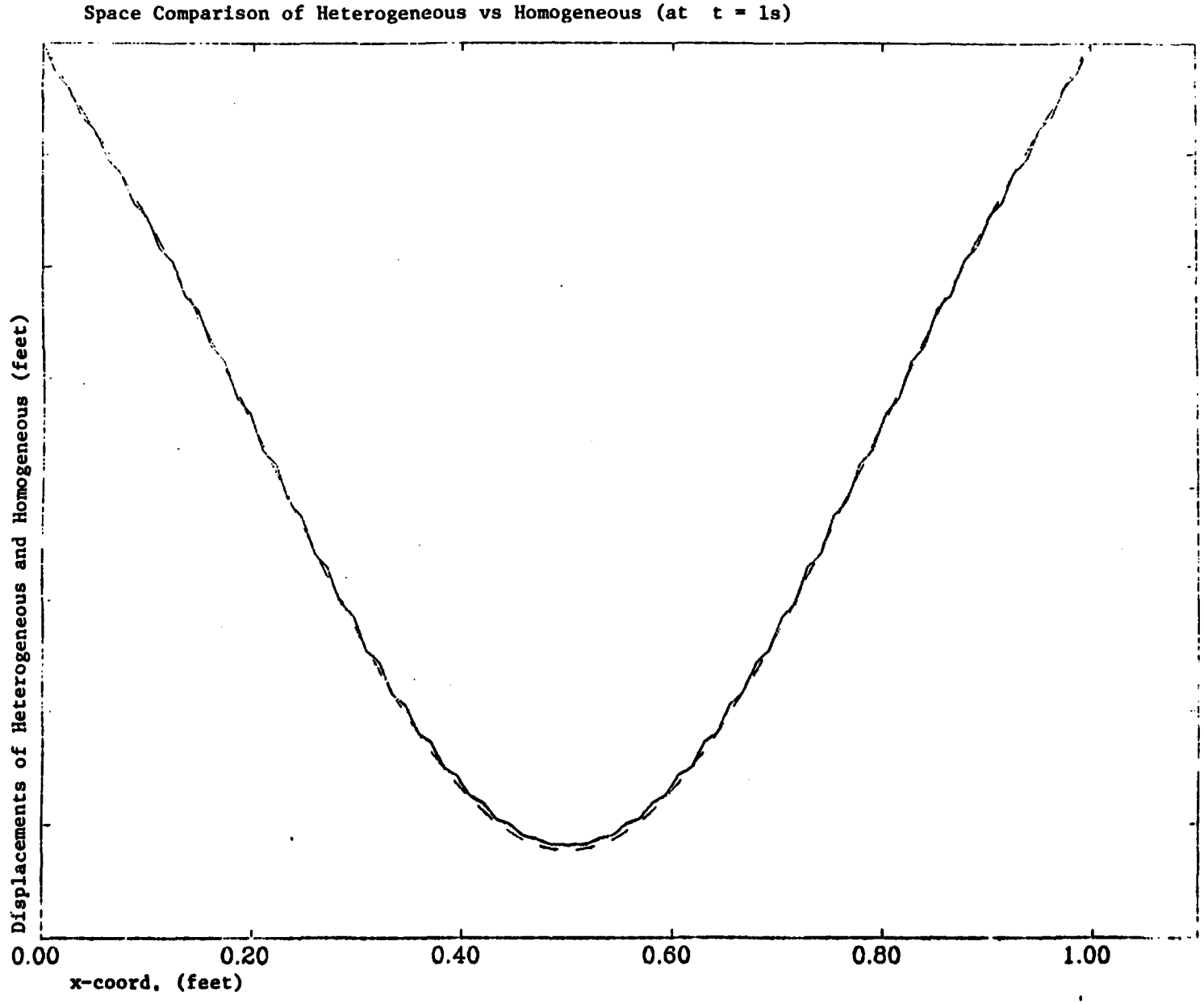
1.0
0.5
0.0
- 0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Time (seconds)

Figure 10


Space Comparison of Heterogeneous vs Homogeneous (at t = 1s)
Displacements of Heterogeneous and Homogeneous (feet)
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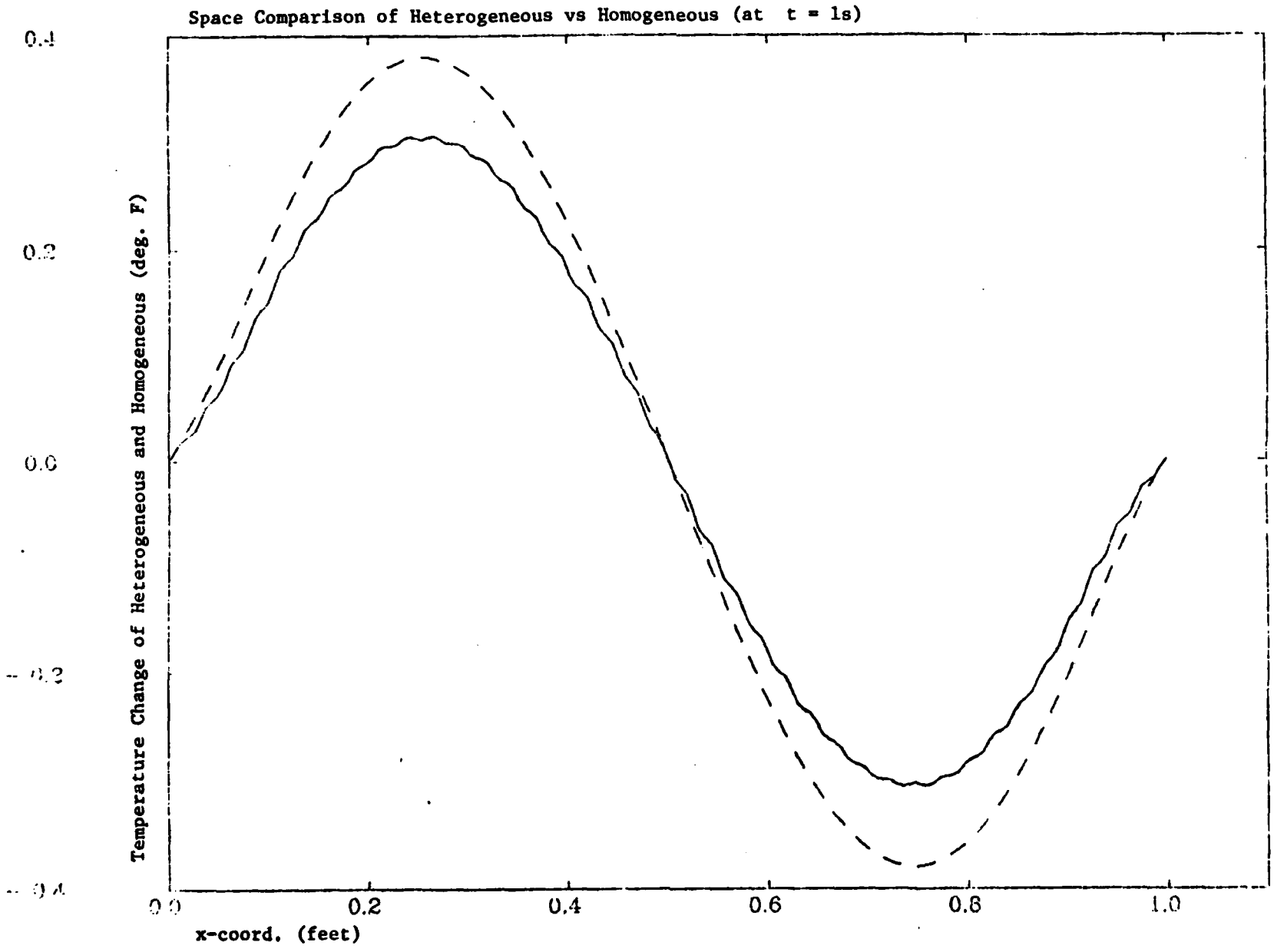
- 0.2
- 0.4
- 0.6
- 1.0
- 1.4
- 1.6
0.00
0.20
0.40
0.60
0.80
1.00
x-coord. (feet)

Figure 11


Space Comparison of Heterogeneous vs Homogeneous (at t = 1s)
Temperature Change of Heterogeneous and Homogeneous (deg. F)
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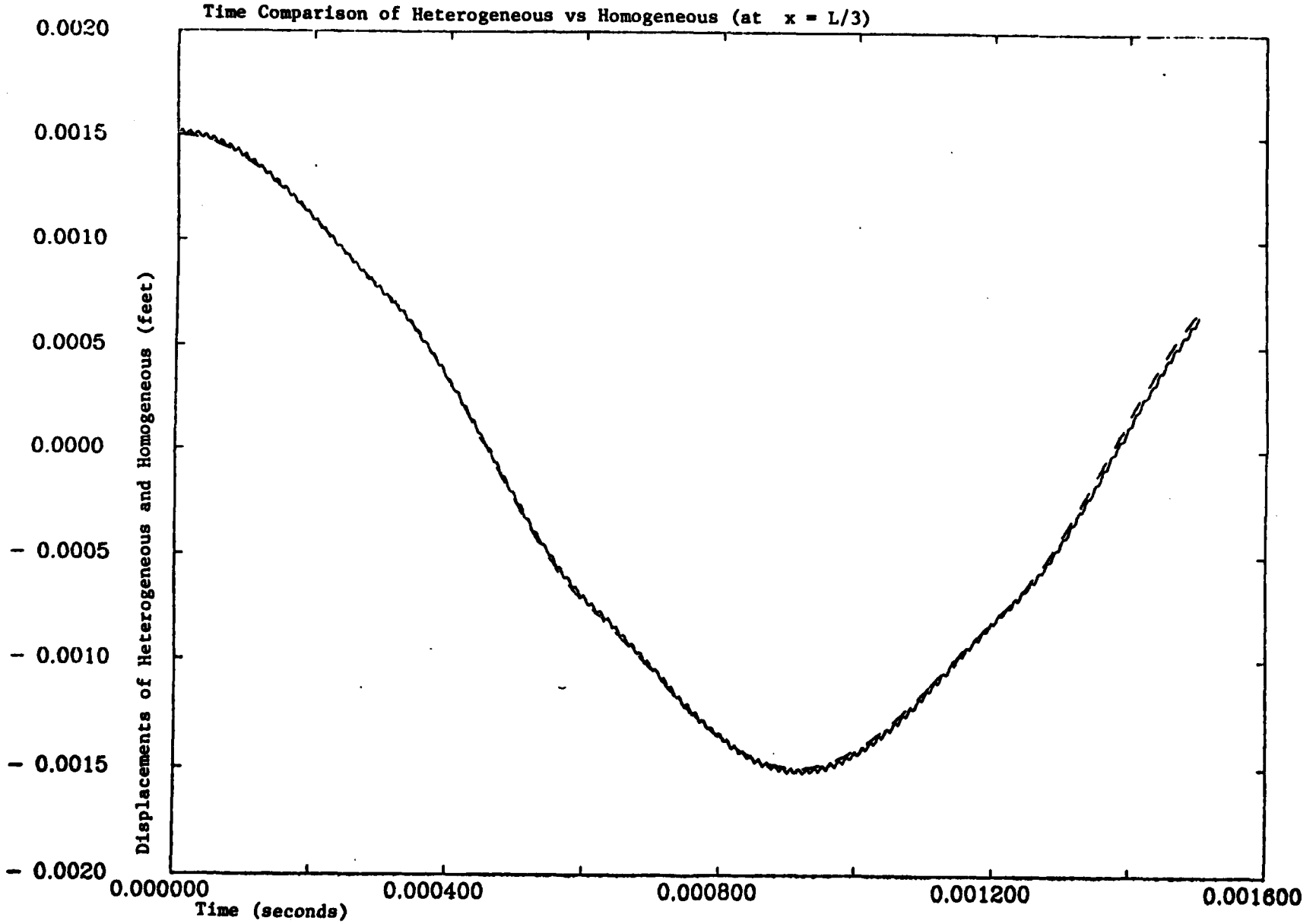
0.2
0.0
0.2
0.4
0.0
0.2
0.4
0.6
0.8
1.0
x-coord. (feet)

Figure 12


Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
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0.0015
0.0010
0.0005
0.0000
- 0.0005
- 0.0010
- 0.0015
- 0.0020
Displacements of Heterogeneous and Homogeneous (feet)
0.000000
0.000400
0.000800
0.001200
0.001600
Time (seconds)

Figure 13

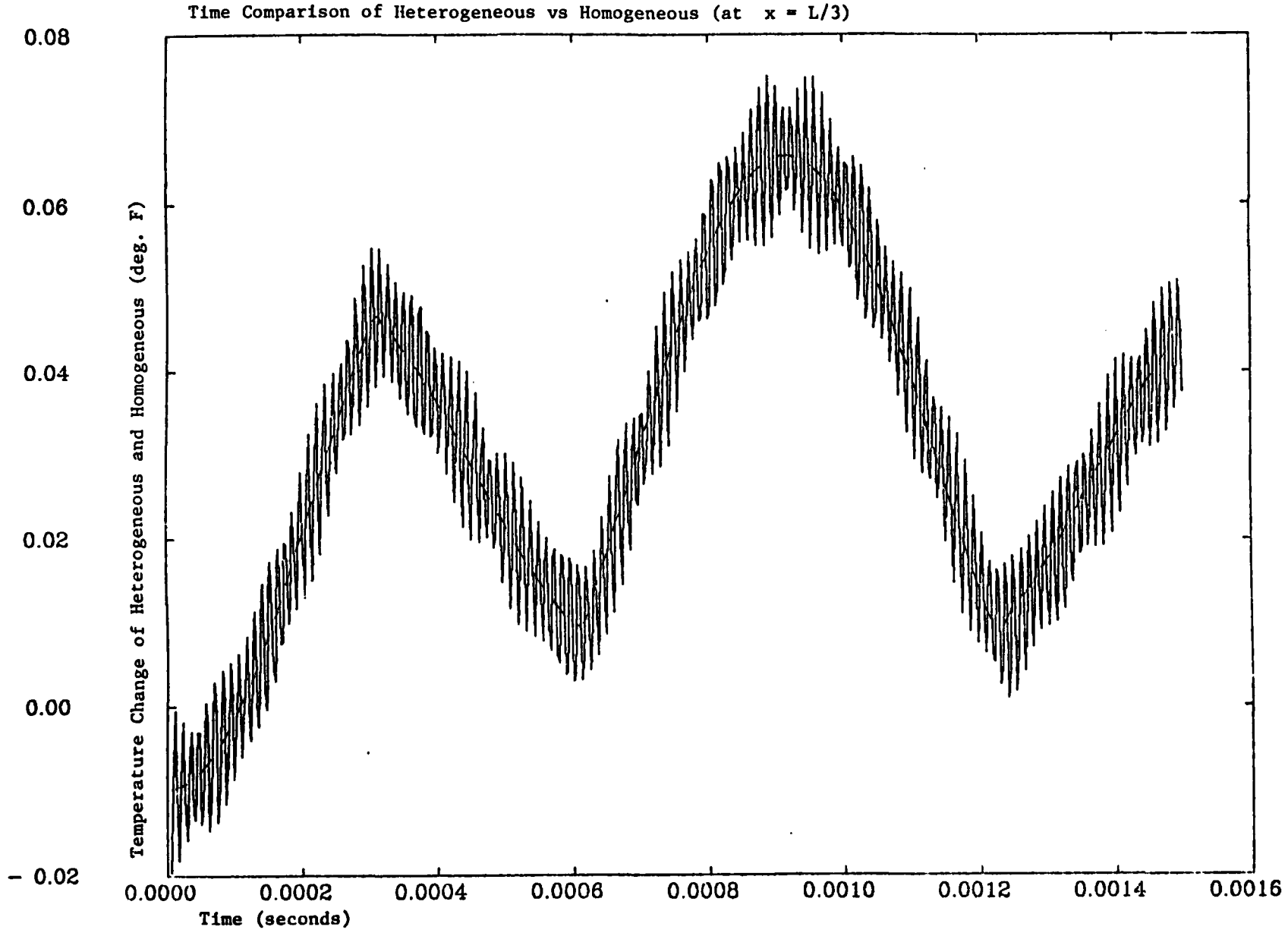
Time Comparison of Heterogeneous vs Homogeneous (at x = L/3)
0.08
0.06
0.04
0.02
0.00
- 0.02
Temperature Change of Heterogeneous and Homogeneous (deg. F)
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012
0.0014
0.0016
Time (seconds)

Figure 14


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7-8